Int. J. Heat Mass Transfer. Vol. 11, pp. 699–707. Pergamon Press 1968. Printed in Great Britain

EQUILIBRIUM AIR STAGNATION POINT BOUNDARY-LAYER CALCULATIONS USING A VARIABLE HEAT OF DISSOCIATION MODEL

LELAND A. CARLSON and ROBERT M. NEREM
The Ohio State University, Columbus, Ohio, U.S.A.

(*Received* 28 *April* 1967 *and in revised form* 3 *August* 1967)

Abstract—Numerical calculations for equilibrium air stagnation point heat transfer have been carried out using the variable heat of dissociation model postulated by Fenster and Heyman. In studying the application of this model, it is noted that there is a discrepancy in the conservation of energy equation as used by Fenster and Heyman and that it is this discrepancy which led to their results being significantly different from those of previous investigations. Present calculations indicate that the use of the correct formulation of the boundary-layer energy equation for a variable heat of dissociation model yields results for equilibrium air that are in basic agreement with those obtained using a constant heat of dissociation model.

NOMENCLATURE

C,	$\bar{c}_p/\bar{c}_{p_w}$;
c_i,	mass fraction of specie i;
c_{p_i},	specific heat per unit mass at constant pressure of specie i;
$\bar{c}_p$,	weighted sum of specie specific heats;
D_i,	binary diffusion coefficient;
f,	$\int_0^\eta f_\eta \mathrm{d}\eta$;
f_η,	dimensionless velocity u/u_e;
h,	enthalpy per unit mass of mixture;
h_i,	perfect gas enthalpy per unit mass of specie i;
h_i^0,	heat evolved in the formation of specie i at 0°K per unit mass;
k,	thermal conductivity;
L,	Lewis number for atom–molecule mixture;
N,	$\rho\mu/\rho_w\mu_w$;
Nu,	Nusselt number;
p,	static pressure;
Pr,	Prandtl number, $\bar{c}_p\mu/k$;
r,	cylindrical radius of body;
Re,	Reynolds number;
T,	absolute temperature;
u,	x component of velocity;
$\mathbf{v}$,	velocity vector;
v,	y component of velocity;
$\dot{w}_i$,	mass rate of formation of specie i per unit volume and time;
x,	distance along surface from leading edge;
y,	distance normal to surface;
z,	dimensionless mass fraction, c_a/c_{ae}.

Greek symbols

β,	$2 \,\mathrm{dln}\, u_e/\mathrm{dln}\, \xi$, equals $\frac{1}{2}$ for stagnation point;
η,	defined by equation (9);
θ,	dimensionless temperature T/T_e;
λ,	defined as $(\mathrm{d}u_e/\mathrm{d}x)_s$ for stagnation point;
μ,	absolute viscosity;
ξ,	defined by equation (9);
ρ,	mass density.

Subscripts

a,	atom;
e,	edge of boundary layer;
i,	ith specie of mixture;
N.	nitrogen;

O, oxygen;
s, stagnation conditions;
w, wall;
∞, free stream;

Superscripts
j, equals -1 for axisymmetric body;
k, equals 1 for axisymmetric body.

1. INTRODUCTION

DURING the past decade, there have been numerous calculations of the heat transfer in dissociated air laminar boundary layers corresponding to conditions encountered during atmospheric entry. Such calculations virtually all involve the numerical solution of the appropriate conservation equations. However, they differ in their respective choice of a model to represent the chemical nature of the boundary layer and in the method used to calculate the thermodynamic and transport properties.

For boundary layers in local chemical and thermodynamic equilibrium, the chemical composition and the associated thermodynamic and transport properties are properties of state; and their calculation may proceed quite independent of the solution of the governing differential flow equations. In this manner complex chemical models, including numerous species and their associated chemical reactions, have been included in equilibrium air laminar boundary-layer calculations [1–2] through the use of tabulated gas properties [4] or correlations thereof [5].

However, for non-equilibrium boundary layers the chemistry is directly coupled to the transport phenomena and must be simultaneously solved for along with the macroscopic fluid properties. In investigating such problems it is expedient, and indeed often necessary from the practical viewpoint of carrying out numerical calculations, to adopt a simplified chemistry model that lends itself to inclusion with the equations of change. Extreme care must be used in the choice of such a model; however, it is obvious that, whatever the choice, it must be one that when applied in equilibrium boundary-layer calculations leads to results which are in reasonable agreement with those obtained using the more complex and "exact" chemical models, e.g. [1–3].

One of the most comprehensive treatments of dissociated air boundary-layers has been that of Fay and Riddell [6] who investigated equilibrium and frozen stagnation point air boundary layers for both fully catalytic and non-catalytic surfaces. In their approach, the chemistry was simplified through the assumption that air is a binary mixture of "air" atoms and "air" molecules having a constant heat of dissociation. Their results indicate that the heat transfer to a fully catalytic surface in the presence of a frozen boundary layer is virtually identical to that associated with an equilibrium boundary layer. It should also be noted that their equilibrium boundary-layer results are in general agreement with calculations using exact thermodynamic and transport properties and with shock tube experimental data [7, 8], thus supporting the validity of their assumed model.

Fenster and Heyman [9, 10], however, have taken issue with the results of Fay and Riddell [6]. They have noted that although oxygen and nitrogen are quite similar insofar as transport properties are concerned, they have quite different molecular binding energies, e.g. that of nitrogen is 9·76 eV while that of oxygen 5·1 eV. Thus, air may not be considered as a binary mixture with regard to the chemical reactions since the recombination of two nitrogen atoms will release almost twice as much energy as the recombination of two oxygen atoms. They have consequently proposed the use of a binary model with a variable heat of dissociation, h_a^0, where

$$\left.\begin{aligned} h_a^0 &= -h_{\mathrm{O}}^0, \qquad c_a < 0{\cdot}2346 \\ h_a^0 &= \frac{0{\cdot}2346(-h_{\mathrm{O}}^0) + (c_a - 0{\cdot}2346)(-h_{\mathrm{N}}^0)}{c_a}, \\ &\qquad 0{\cdot}2346 \leqslant c_a \leqslant 1{\cdot}0. \end{aligned}\right\} \quad (1)$$

Here c_a is the atom mass fraction, h_{O}^0 and h_{N}^0 the heats of formation of O and N respectively, and h_a^0 the effective atom heat of formation. The use of such a variable heat of dissociation model appears to the present authors to be a logical extension of the model used by Fay and Riddell. In the limiting case of equilibrium air, nitrogen becomes dissociated at a much higher temperature than oxygen. Thus, in the outer part of an equilibrium boundary layer where the nitrogen atoms recombine, equation (1) will weight the effective heat of formation towards that of nitrogen; while in the cool region near the wall, h_a^0 will be equal to that of oxygen. On the other hand, for the limiting case of a frozen flow, the effective heat of formation will remain constant across the boundary layer since there is no recombination in the flow itself. In this case equation (1) may be applied at the edge of the boundary layer to evaluate the constant heat of dissociation value.

In applying this model to the stagnation point boundary layer, Fenster and Heyman found that in the presence of dissociation the surface heat transfer associated with an equilibrium air boundary layer was less, in some cases significantly, than that associated with a frozen boundary layer on a catalytic surface. This difference from the results of Fay and Riddell was due to their prediction of much lower heat transfer for highly dissociated equilibrium boundary layers and the resulting difference in the dependency of the non-dimensional heat-transfer parameter, $[Nu/\sqrt{(Re)}]_w$ on the ratio of the density–viscosity product, $(\rho_e\mu_e/\rho_w\mu_w)$. Whereas, Fay and Riddell predicted a power dependence of 0·4, Fenster and Heyman in using a variable heat of dissociation model predicted a power dependence of 0·595. Unfortunately, it is the former value that is in general agreement with a majority of the "exact" equilibrium air boundary-layer calculations [1–3]. Thus, the concept of a variable heat of dissociation model would appear somewhat questionable.

However, the present authors, in studying the application of the variable heat of dissociation model, have noted what they feel is a discrepancy in the conservation of energy equation as used by Fenster and Heyman in their equilibrium boundary-layer calculations. They have further shown that it is this discrepancy that leads to the differences between the results obtained using a constant heat of dissociation model and those of Fenster and Heyman. Thus, in the next section the energy equation for a gas with a variable heat of dissociation will be derived; while in Section 3 the numerical solution of the present authors will be discussed. The results obtained for equilibrium air stagnation point boundary layers and their implications will be presented in Section 4.

2. THE EQUATIONS OF CHANGE FOR A VARIABLE HEAT OF DISSOCIATION

The equations of change which describe the steady laminar boundary-layer flow of a chemically reacting gas are [6], neglecting thermal diffusion,

$$\frac{\partial}{\partial x}(\rho u r^k) + \frac{\partial}{\partial y}(\rho v r^k) = 0 \tag{2}$$

$$\rho u \frac{\partial c_i}{\partial x} + \rho v \frac{\partial c_i}{\partial y} = \frac{\partial}{\partial y}\left(\rho D_i \frac{\partial c_i}{\partial y}\right) + \dot{w}_i \tag{3}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{4}$$

$$\frac{\partial p}{\partial y} = 0$$

$$\begin{aligned}\rho \mathbf{v}\,.\,\nabla h &= \rho u \frac{\partial h}{\partial x} + \rho v \frac{\partial h}{\partial y}\\ &= \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + u \frac{\partial p}{\partial x} + \mu\left(\frac{\partial u}{\partial y}\right)^2\\ &\quad + \frac{\partial}{\partial y}\left[\sum_i D_i \rho \frac{\partial c_i}{\partial y}(h_i - h_i^0)\right].\end{aligned} \tag{5}$$

If the mixture enthalpy is defined as

$$h = \sum_i c_i(h_i - h_i^0) \tag{6}$$

where both h_i and h_i^0 are assumed to be variable and functions of temperature alone, then the static enthalpy gradient may be written as

$$\nabla h = \sum_i (h_i - h_i^0)\,\nabla c_i + \bar{c}_p \nabla T - \sum_i c_i \nabla h_i^0 \qquad (7)$$

where

$$\bar{c}_p = \sum_i c_i \frac{\mathrm{d}h_i}{\mathrm{d}T} = \sum_i c_i c_{p_i}.$$

Subsequently, the energy equation may, upon substitution of the species continuity equation, be written as

$$\begin{aligned}\bar{c}_p \left(\rho u \frac{\partial T}{\partial x} + \rho v \frac{\partial T}{\partial y}\right) &= \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + u \frac{\partial p}{\partial x} \\ &+ \mu \left(\frac{\partial u}{\partial y}\right)^2 - \sum_i \dot{w}_i (h_i - h_i^0) \\ &+ \sum_i \left[D_i \rho \frac{\partial c_i}{\partial y}\left(c_{p_i} \frac{\partial T}{\partial y} - \frac{\partial h_i^0}{\partial y}\right)\right] \\ &+ \left\{\rho u \sum_i c_i \frac{\partial h_i^0}{\partial x} + \rho v \sum_i c_i \frac{\partial h_i^0}{\partial y}\right\}. \qquad (8)\end{aligned}$$

Equation (8) differs from the expression originally obtained by Fenster and Heyman [10] for the variable heat of dissociation model in that it contains the additional term

$$\left\{\rho u \sum_i c_i \frac{\partial h_i}{\partial x} + \rho v \sum_i c_i \frac{\partial h_i}{\partial y}\right\}.$$

This additional term arises because the present authors have applied the variable heat of dissociation concept starting with the energy equation written in terms of the static enthalpy, whereas Fenster and Heyman apparently applied the same concept, but starting with the energy equation in terms of temperature (it should be noted that Fenster and Heyman could also have arrived at their form of the energy equation through erroneous differentiation). Since in deriving the energy equation, it is the static enthalpy, or internal energy, that naturally appears and not the temperature, it is felt by the present authors that equation (8) is the correct equation for a gas with a variable heat of dissociation. Furthermore, an analysis of this additional term indicates that it is of the same magnitude and sign as the term

$$\left(-\sum_i D_i \rho \frac{\partial c_i}{\partial y}\frac{\partial h_i^0}{\partial y}\right).$$

Thus, it is felt that it should be included in any solution using the variable heat of dissociation concept.

Now if local similarity and a binary gas mixture are assumed, then with the coordinate transformation

$$\left.\begin{aligned}\xi(x) &= \int_0^x \rho_w \mu_w u_e r^{2k}\,\mathrm{d}x \\ \eta(x, y) &= \frac{u_e r^k}{\sqrt{2\xi}} \int_0^y \rho\,\mathrm{d}y\end{aligned}\right\} \qquad (9)$$

the equations of change become

$$\frac{\partial}{\partial \eta}\left(\frac{NL}{Pr}\frac{\partial z_i}{\partial \eta}\right) + f \frac{\partial z_i}{\partial \eta} + \frac{2^j \lambda^{-1} \dot{w}_i}{\rho c_{i_e}} = 0 \qquad (10)$$

$$(Nf'')' + ff'' + \beta\left(\frac{\rho_e}{\rho} - f'^2\right) = 0 \qquad (11)$$

$$\begin{aligned}&\left(\frac{CN\theta'}{Pr}\right)' + Cf\theta' + \frac{Nu_e^2}{\bar{c}_{p_w} T_e}(f'')^2 \\ &\quad + \sum_i \frac{2^j \lambda^{-1} \dot{w}_i (h_i^0 - h_i)}{\rho \bar{c}_{p_w} T_e} + \sum_i \frac{NL}{\bar{c}_{p_w} Pr} \\ &\quad \times c_i' \left(c_{p_i} \theta' - \frac{h_i^{0\prime}}{T_e}\right) - \frac{f \sum_i c_i h_i^{0\prime}}{\bar{c}_{p_w} T_e} = 0. \qquad (12)\end{aligned}$$

where the non-dimensional variables are as defined in the Nomenclature and the summation over i involves only the constituents of the binary gas—atoms and molecules. As in equation (8), the last term on the l.h.s. of equation (12) was not included in the original derivation by Fenster and Heyman.

It should be noted that in applying the above equation to the equilibrium laminar boundary layer, it is convenient to eliminate the term involving the mass rate of formation of species i, $\dot{w}_i$, from the energy equation. This may be done by solving the specie continuity equation, equation (10), for $2^j\lambda^{-1}\dot{w}_i/\rho$ and substituting into the energy equation. The resulting equation is

$$\left(\frac{CN\theta'}{Pr}\right)' + Cf\theta' + \frac{Nu_e^2}{\bar{c}_{p_w}T_e}f''^2 + \sum_i \frac{c_{i_e}(h_i - h_i^0)}{\bar{c}_{p_w}T_e}\left[fz_i' + \left(\frac{NLz_i'}{Pr}\right)'\right] + \sum_i \frac{NL}{\bar{c}_{p_w}Pr}c_i'\left(c_{p_i}\theta' - \frac{h_i^{0\prime}}{T_e}\right) - \frac{f\sum c_i h_i^{0\prime}}{\bar{c}_{p_w}T_e} = 0. \tag{13}$$

It should be noted that the derivatives of h_i^0 in the above equation are actually of an ad-hoc nature due to the empirical character of equation (1). They result solely from the use of a variable heat of dissociation to characterize the single reaction present in a binary mixture of air molecules and atoms.

3. NUMERICAL SOLUTION

In the present investigation, calculations have been carried out for the equilibrium stagnation point boundary layer using a variable heat of dissociation as given by equation (1) together with equations (11) and (13) which were discussed in the previous section. In these solutions, the Prandtl number was assumed to be constant at 0·71. The density–viscosity product ratio was evaluated using the correlation of Penner and Litvak (see Fay and Riddell); and the equilibrium atomic mass fraction was approximated by $c_a = c_{a_e} \exp[\alpha(1 - 1/\theta)]$, where the constant α was selected in each case to yield values of c_a that agreed closely with the tables of Hilsenrath and Klein [11]. The ratios ρ_e/ρ and $\bar{c}_p/\bar{c}_{p_w}$ were evaluated as in [6], and the wall temperature was assumed to be 300°K. Calculations were carried out for both the case of a constant Lewis number, i.e. $L = 1{\cdot}0$ and $1{\cdot}4$, and for a variable Lewis number. In this latter case, the Lewis number was evaluated as a function of the atom concentration so as to be approximately that given in [4]. Thus, the thermodynamic and transport properties used here are virtually identical to those used by Fenster and Heyman; and the primary difference between the present results and those of [10] will be due to the form of the energy equation used.

As has been discussed, the energy equation used by Fenster and Heyman did not include the term $(-f\sum_i c_i h_i^{0\prime}/\bar{c}_{p_w}T_e)$. Furthermore, in most of their actual numerical solutions they neglected in equation (13) the term

$$-\sum_i \frac{NLc_i'}{\bar{c}_{p_w}Pr}\frac{h_i^{0\prime}}{T_e}$$

noting that its overall effect was small. However, this latter term and the last term in equation (13) are of the same order of magnitude; and as will be seen, their combined effect need not be small.

In the present investigation equations (11) and (13) were solved simultaneously using a fourth order Runge–Kutta technique subject to the following boundary conditions:

$$\left.\begin{aligned} &f'(0) = f(0) = 0, \qquad \theta(0) = \frac{T_w}{T_e}, \\ &f'(\infty) = 1, \qquad \theta(\infty) = 1. \end{aligned}\right\} \tag{14}$$

The solutions were obtained by iterating on the initial values of $\theta'(0)$ and $f''(0)$ until the conditions at "infinity" were satisfied. The actual iteration technique is described in [6, 10]. Solutions were obtained for three cases. These are:

Case 1. The terms,

$$-f\sum_i c_i h_i^{0\prime}/\bar{c}_{p_w}T_e \text{ and } -\sum_i \frac{NL}{\bar{c}_{p_w}Pr}c_i'\frac{h_i^{0\prime}}{T_e}$$

omitted in equation (13).

Case 2. The term, $-f\sum_i c_i h_i^{0\prime}/\bar{c}_{p_w}T_e$ omitted in equation (13).

Case 3. All terms in Equation (13) included.

Case 1 corresponds to that for which solutions were obtained by Fenster and Heyman.

4. DISCUSSION OF RESULTS

A summary of the numerical results obtained in the present investigation is presented in Figs. 1–3, and a portion of the calculated data are listed in Table 1. In Fig. 1 values of the heat-transfer parameter, $(Nu/\sqrt{Re})_w$, computed at a specific flight condition using a variable heat of dissociation and both constant and variable Lewis numbers, are compared for cases 1–3. From the figure, it may be seen that in both cases 1 and 3, the $(Nu/\sqrt{Re})_w$ obtained using a variable Lewis number differs from that for a constant Lewis number ($L = 1{\cdot}4$) by, at the most, 3 per cent. Furthermore, for a constant Lewis number the inclusion of $\sum_i c_i' h_i^{0\prime}/T_e$ in the energy equation, i.e. case 2, results in an increase in the $(Nu/\sqrt{Re})_w$ of 9·0 per cent over that for case 1, which corresponds to that used in the original results of Fenster and Heyman.

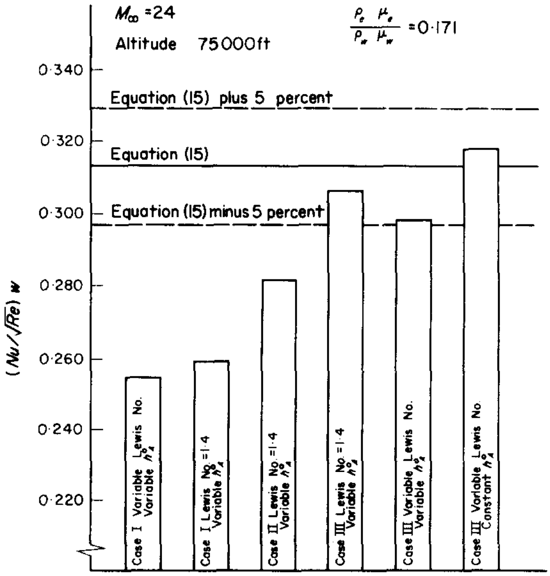

FIG. 1. Comparison of stagnation point heat-transfer results for various dissociated air models (cases 1–3).

Table 1. Summary of numerical results

M_∞	Altitude (ft)	L	$f''(0)$	$\theta'(0)$	$\frac{\rho_e\mu_e}{\rho_w\mu_w}$	$\frac{Nu}{\sqrt{(Re)}}$
Case 1						
10·00	75000	1·4	0·476	0·396	0·424	0·424
16·53	80000	1·4	0·391	0·508	0·262	0·366
20·66	80000	1·4	0·357	0·534	0·204	0·300
24·00	75000	1·4	0·336	0·552	0·171	0·259
24·00	75000	1·0	0·339	0·518	0·171	0·243
24·00	75000	variable	0·337	0·542	0·171	0·254
26·86	80000	1·4	0·317	0·578	0·140	0·233
Case 2						
24·00	75000	1·4	0·341	0·601	0·171	0·282
Case 3						
10·95	150000	variable	0·462	0·564	0·344	0·431
12·77	150000	variable	0·421	0·567	0·295	0·400
16·53	80000	variable	0·391	0·509	0·262	0·367
20·66	80000	variable	0·365	0·591	0·204	0·332
21·89	150000	variable	0·346	0·779	0·161	0·292
24·00	75000	1·0	0·347	0·604	0·171	0·284
24·00	75000	1·4	0·345	0·653	0·171	0·307
24·00	75000	variable	0·346	0·633	0·171	0·297
26·86	80000	variable	0·326	0·681	0·140	0·275
Constant h_a^0						
16·53	80000	variable	0·392	0·536	0·262	0·386
20·66	80000	variable	0·361	0·638	0·204	0·358
24·00	75000	variable	0·341	0·677	0·171	0·318
26·86	80000	variable	0·321	0·760	0·140	0·307

The inclusion of both terms, i.e. case 3, results in an 18·4 per cent increase over that calculated for case 1. These increases indicate that the two additional terms in equation (12) are indeed both significant.

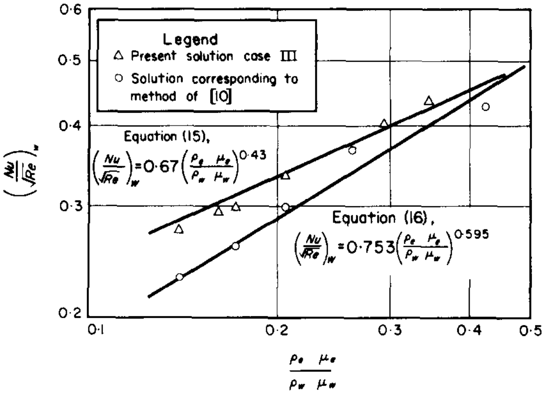

FIG. 2. Dependence of heat-transfer parameter, $Nu/Re^{\frac{1}{2}}$, on $\rho\mu$-ratio for dissociated air models of cases 1 and 3.

Shown also in Fig. 1 for the same flight condition is the value obtained using a constant heat of dissociation with a variable Lewis number. Comparison of this result with that obtained using a variable heat of dissociation (case 3 with a variable Lewis number) indicates that the two answers are in reasonable agreement, being within 7 per cent of each other.

Also indicated in Fig. 1 is the value of the $(Nu/\sqrt{Re})_w$ obtained using Cohen's correlation, i.e.

$$\left(\frac{Nu}{\sqrt{Re}}\right)_w = 0{\cdot}67\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0\cdot43} \tag{15}$$

of his numerical calculations of equilibrium dissociated air stagnation point heat transfer. These calculations were obtained using the thermodynamic and transport properties of [4]. As can be seen from Fig. 1, the present case 3 calculations are within ±5 per cent of Cohen's results.

The stagnation point heat-transfer rate was also calculated for cases 1 and 3 over a range of values of the density–viscosity product ratio, $\rho_e\mu_e/\rho_w\mu_w$. These results are presented in Fig. 2. Although the case 1 calculations are for a constant Lewis number of 1·4 and those for case 3 are for a variable Lewis number, it was shown in Fig. 1 that the effect of Lewis number on the calculated heat transfer is small. Included here is the correlation curve of Fenster and Heyman

$$\left(\frac{Nu}{\sqrt{Re}}\right)_w = 0{\cdot}753\left(\frac{\rho_e\mu_e}{\rho_w\mu_w}\right)^{0\cdot595}. \tag{16}$$

It can be seen that the present case 1 calculations, which corresponds to the method used in [10], are in reasonable agreement with equation (16). However, the case 3 calculations, based on the complete formulation of the boundary-layer equations for a variable heat of dissociation model, are significantly higher than the results of Fenster and Heyman, equation (16), and are in good agreement with the correlation of Cohen's results, equation (15). Thus the present calculations indicate a power dependence of the heat-transfer parameter on the $\rho\mu$ ratio of approximately 0·4. As noted in the introduction, this is in good agreement with a majority of the existing exact equilibrium air boundary-layer calculations.

Calculations using a constant heat of dissociation model have also been carried out in the present investigation over a range of conditions and these are compared with Case 3 results in Fig. 3. As can be seen, the calculations based on a

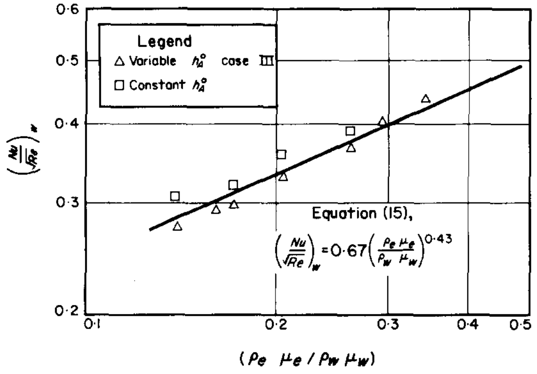

FIG. 3. The effect of constant vs. variable heat of dissociation on stagnation point heat transfer (variable Lewis number).

constant heat of dissociation model are no more than 10 per cent higher than those obtained using a variable heat of dissociation model.

It should be noted that as a further check on the accuracy of the present method of numerical solution, several calculations were made using a constant heat of dissociation and the input data of Fay and Riddell [6]. The results of these calculations are in good agreement with those reported in [6].

5. CONCLUSIONS

The present investigation has shown that the correct formulation of the boundary-layer energy equation using a variable heat of dissociation model leads to results for equilibrium air which are in basic agreement with those obtained using a constant heat of dissociation model.

These results, i.e. case 3, are also in agreement with exact equilibrium air boundary-layer calculations [1–3] and can be shown to agree with shock tube stagnation point heat-transfer measurements over a wide range of conditions [7, 8]. This is a rather secondary point, however, since the results of Fenster and Heyman have been shown to be in reasonable agreement with an exact calculation [12] using the transport properties of Peng and Pindroh [13] and with the limited shock tube data of Sabol [14], which were obtained at relatively low total enthalpies.

The important point is that in any application of the variable heat of dissociation model, the natural form of the energy equation in terms of static enthalpy or internal energy, and not temperature, must be considered as the basic underlying equation. This was done here, but apparently not by Fenster and Heyman; and the results are significantly different. The calculations and conclusions of Fenster and Heyman thus must be considered in error. Furthermore, the extension of the variable heat of dissociation concept to ionized air boundary layers by Fenster [15] is similarily in error; and those results must be also considered questionable.

ACKNOWLEDGEMENTS

This research was supported by the Aerospace Research Laboratories, Office of Aerospace Research, USAF, Wright–Patterson Air Force Base, Ohio under Contract AF 33(615)-2220; and monitored by the Air Force Office of Scientific Research.

REFERENCES

1. N. B. Cohen, Boundary-layer solutions and correlation equations for laminar heat-transfer distribution in equilibrium air up to 41 000 feet per second, NASA TR R-118 (1961).
2. H. Hoshizaki, Heat transfer in planetary atmospheres at super-satellite speeds, *ARS Jl* **22**, 1544–1552 (1962).
3. A. Pallone and W. Van Tassell, Effects of ionization on stagnation point heat transfer in air and nitrogen, *Physics Fluids* **6**, 983–986 (1963).
4. C. F. Hansen, Approximations for the thermodynamic and transport properties of high temperature air, NASA TR R-50 (1959).
5. N. B. Cohen, Correlation formulas and tables of density and some transport properties of equilibrium dissociating air for use in solutions of the boundary-layer equations, NASA TN D-194 (1960).
6. J. A. Fay and F. R. Riddell, Theory of stagnation point heat transfer in dissociated air, *J. Aero/Space Sci.* **25**, 78–85 (1958).
7. P. H. Rose and J. O. Stankevice, Stagnation point heat-transfer measurements in partially ionized air, *AIAA Jl* **1**, 2752–2763 (1963).
8. R. M. Nerem and G. H. Stickford, Radiative and convective heating during hypervelocity re-entry, *AIAA Jl* **2**, 1156–1158 (1964).
9. S. J. Fenster and R. J. Heyman, Effects of variable heat of recombination on stagnation point heat transfer, *J. Aerospace Sci.* **29**, 1267 (1962).
10. S. J. Fenster and R. J. Heyman, Heat transfer in dissociated air with variable heat of dissociation, *Int. J. Heat Mass Transfer* **6**, 1063–1075 (1963).
11. J. Hilsenrath and M. Klein, Tables of thermodynamic properties of air in chemical equilibrium including second virial corrections from 7500°K to 15000°K, AEDC-TR-65-58, Arnold Engineering Development Center, Tennessee (1965).
12. R. C. Rozycki and S. J. Fenster, Stagnation-point heat transfer in partially ionized air, *J. Aerospace Sci.* **29**, 1490 (1962).
13. T. C. Peng and A. L. Pindroh, An improved calculation of gas properties at high temperatures: air, Boeing Airplane Co., Doc. D2-11722 (1962).
14. A. B. Sabol, Measurements in a shock tube of heat-transfer rates in the stagnation point of a 1·0 inch diameter sphere for a real-gas temperature up to 7900°K, NACA Report TN-4354 (1958).
15. S. J. Fenster, Stagnation-point heat transfer for a new binary air model including dissociation and ionization, *AIAA Jl* **3**, 2189–2196 (1965).

Résumé—Des calculs numériques pour le transport de chaleur au point d'arrêt dans l'air en équilibre ont été conduits en employant le modèle à chaleur de dissociation variable imaginé par Fenster et Heyman. En étudient l'application de ce modèle, on remarquera qu'il y a une différence dans l'équation de conservation de l'énergie employée par Fenster et Heyman et que c'est cette différence qui conduit à ce que leurs résultats sont sensiblement différents de ceux des recherches antérieures. Les calculs actuels indiquent que que l'emploi de la formulation de l'équation de l'énergie de la couche limite pour un modèle à chaleur de dissociation variable fournit des résultats pour l'air en équilibre qui sont en accord général avec ceux obtenus avec un modèle à chaleur de dissociation constante.

Zusammenfassung—Numerische Berechnungen wurden durchgeführt für den stationären Wärmeübergang in Luft an einem Staupunkt unter Benützung eines Modells veränderlicher Dissoziationswärme nach Fenster–Heyman. Bei der Untersuchung der Anwendbarkeit dieses Modells wurde ein Widerspruch in der von Fenster–Heyman verwendeten Form der Energiegleichung bemerkt und es zeigt sich, dass auf diesem Widerspruch die wesentlichen Unterschiede ihrer Ergebnisse zu denen früherer Forscher beruhten. Die gegenwärtigen Rechnungen ergeben, dass bei korrekter Formulierung der Grenzschichtenergiegleichung für das Modell veränderlicher Dissoziationswärme die Ergebnisse für Luft grundsätzlich mit jenen übereinstimmen, die mit einem Modell konstanter Dissoziationswärme gefunden wurden.

Аннотация—С помощью модели переменной теплоты диссоциации Фенстера и Хеймена проведены численные расчеты теплообмена в критической точке равновесного течения воздуха. При этом замечено, что имеется различие в уравнениях энергии, использованных нами и Фенстером, и Хейманом. Оно было причиной значительного отличия результатов последних авторов от данных других исследователей. Наши расчеты показывают, что при использовании правильной формулировки уравнения энергии пограничного слоя и модели переменной теплоты диссоциации для равновесного течения воздуха, получаются результаты, в основном совпадающие с данными, полученными при использовании модели постоянной теплоты диссоциации.